Advancing the Combination-Permutation Algorithm [1]

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Introduction

Since my initial article on the Combination-Permutation Algorithm (CPA) for portfolio allocation [1] was published, I have continued to work on this issue.

Building on the previous formulation [1], this article presents the "new and improved formulation" of the algorithm. It also explains the rationale behind CPA, the possibilities for its implementation, and the research it enables.

The Rationale Behind CPA

The typical approach to portfolio allocation involves sequential steps such as project prioritization, selection, scheduling, sequencing, and staging to avoid resource contention. The exact steps and their sequence may vary, and some of them are sometimes combined, but in any case, this approach has inherent fundamental problems.

According to this approach, the portfolio goal (such as maximizing profit) can be achieved by making the best decisions at each step of the process. In other words, the resource allocation problem is broken down into subproblems that are solved separately. It seems logical that the best solutions to each subproblem should provide the best solution to the overall problem.

However, the portfolio goal depends on multiple factors that must be taken into account in their entirety at the same time. The very first step in the typical process leads to a suboptimal solution that cannot be corrected in subsequent steps. Each following step results in a suboptimal solution within an already limited, suboptimal region of possible solutions. Thus, the suboptimality accumulates.

The sum of local optimizations is not equivalent to global optimization.

Some computer games tempt players with large prizes to try to distract them from the goal of the game. Similarly, each step in the allocation of a project portfolio tempts with the prize of the best

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solution to the corresponding subproblem. This is a classic *greedy trap.* Nonetheless, if the excitement of the portfolio allocation game is more important than the goal, we can ignore the outcome and enjoy the process.

Let's assume that the first step of the allocation limits the maximum portfolio profit to 90% of the theoretically possible one. The second and third steps limit the maximum profit to 90% of the maximum that can be achieved after the previous step. Thus, the three steps limit the portfolio profit to 73% of the maximum possible one. So far, we have generated systemic waste (waste of local optimization) of 27% of the maximum possible portfolio profit.

What Makes the Combination-Permutation Algorithm Different?

In short, CPA performs all traditional steps of project portfolio allocation **simultaneously**, taking into account all defined conditions and restrictions. Instead of a series of local decisions, the algorithm involves a single global decision that, within the defined parameters, ensures optimal allocation.

CPA redefines the allocation problem: from a series of interrelated subproblems to a single-goal optimization problem. The algorithm identifies all feasible "global portfolio configurations." The one that maximizes the portfolio's profit is selected, and it determines the allocation of resources.

Each global configuration is a specific variation combining a set of projects prioritized by completion order, project schedules, and a portfolio schedule. Selection, sequencing, and staging are implicit in global configurations. They contain all the information about the expected benefits, costs, and profit of the portfolio. Allocation is reduced to selecting that ready-to-execute configuration that maximizes the portfolio's profit.

The process of initial portfolio planning is inseparable from the resource allocation process, and they are carried out simultaneously.

CPA can be applied with both deterministic and stochastic input data. The latter case requires the integration of a Monte Carlo simulation.

The algorithm takes into account all conditions and restrictions that portfolio management deems necessary, including financial restrictions, scheduling constraints, project cost-of-time profiles [1], resource availability, and external and inter-project dependencies (including synergies).

The inherent variability in the portfolio is reflected in the input values of the stochastic parameters that affect the portfolio’s benefits and costs.

The impact of discrete risks is modeled by varying certain stochastic parameters that are affected. For example, an accidental interruption of a transport route can be accounted for by extending the delivery time (e.g., from 15-25 days under normal conditions to 25-40 days). The risk probability can also be defined stochastically.

The level of confidence in achieving a certain portfolio profit (e.g., 80%), as well as the budget confidence level (e.g., 95%), are treated as decision variables. These levels may differ. The optimal global portfolio configuration is the one that achieves the highest profit at the desired level of confidence.

The New Formulation

The new formulation of the CPA has two main improvements.

First, the portfolio constraints (in the sense of the Theory of Constraints) no longer need to be known in advance. In fact, the constraints may be different for different global configurations, and they are identified after the optimal configuration is chosen.

And second, the improved algorithm works with stochastic input and output data.

The new formulation of the Combination-Permutation Algorithm includes the following stages:

1. **Defining and identifying input data**: a list of potential projects, project benefits and costs data, profit calculation models, required and available resources, base project schedules, stochastic parameters and discrete risks, as well as their values and distribution types, external and inter-project dependencies, cost-of-time profiles, and any conditions and restrictions.

2. **Identifying all project combinations** (A, B, C, AB, AC, ABC, etc.). The infeasible combinations are eliminated and no longer considered.

3. **Detecting all permutations** for each feasible combination. For example, for the combination ABC, the permutations are A-B-C, A-C-B, B-A-C, B-C-A, etc. The sequence indicates the order of completion of the projects. The infeasible permutations are removed and not considered further.

4. **Generating all feasible variations of project schedules and a portfolio schedule** for each feasible permutation. Doing this, resource conflicts are resolved, and all defined conditions and restrictions are respected. The feasible variations of the portfolio schedule represent the feasible "global configurations" of the portfolio.

5. **Deriving a probability distribution of portfolio profit**. This is performed for each feasible configuration through Monte Carlo analysis, which uses the values as well as the distribution types of the input parameters. Probability distributions of benefits, total costs, and investment costs are derived as well.

6. **Selecting the configuration with the highest portfolio profit** at the desired level of confidence. Before comparing the configurations, those that exceed the portfolio budget at the desired level of budget confidence are eliminated.

7. **Identifying the constraint(s)** of the selected portfolio configuration, in the sense of the Theory of Constraints.

Within the modeled decision space, CPA provides optimal resource allocation and an optimized and feasible portfolio execution plan. Although the algorithm is presented in separate stages, these are not steps that lock suboptimal solutions, but ones that prepare and perform an exhaustive combinatorial search for the optimal solution.

The deterministic profit is not used to filter global configurations, even when it is negative, because the stochastic profit can be positive, nonetheless.

Monte Carlo analysis does not verify or ensure the feasibility of configurations, but through profit variation "punishes" those that are unstable under stochastic conditions.

Although project and portfolio schedule variations are combinatorially explosive, as is known, they are a finite number, since the scheduling parameters are a finite number.

CPA has deterministic, stochastic, and optimization layers. The optimization component is reduced to a simple action: choosing the configuration with the highest portfolio profit. Thanks to this, the algorithm not only works at a conceptually superior level compared to existing methods but also simplifies and objectifies the decision-making process for allocating portfolio resources.

Applicability of the Combination-Permutation Algorithm

Let us consider two important aspects of the algorithm's applicability: the aspect related to the portfolio goal and the need for computational resources.

CPA is goal-neutral. It can be applied to any portfolio goal, as long as it is the only one for the portfolio or, in the case of multiple goals, they are expressed in a common measure.

It can also be applied to goals expressed in non-monetary units, such as Value-Cost Units (VCUs) or Benefit-Cost Units (BCUs).

VCUs measure the value in a project or portfolio directly through the metrics of benefits, such as hours of free time or the number of lives saved. Costs are converted to the same units used for value so that they are consistent. To illustrate, if benefits and costs are expressed in hours of free time, 900 h (VCUs, value) minus 500 h (VCUs, cost) equals 400 h (VCUs, profit).

The conversion can be done in various ways. One way is to use the cost-benefit ratio of the best alternative investment. If the best alternative investment generates one unit of benefit for an investment of $10K, then each unit of cost for the project under consideration is equal to $10K. A project with an investment of $1M will have 100 VCUs of investment costs.

Regarding the computational resource requirements, except for small portfolios with low uncertainty, the full algorithm cannot be implemented with standard software tools. To illustrate the challenge, 15 projects have more than 1.3 trillion permutations, and the number of permutations for all combinations of 15 projects exceeds 3.5 trillion.

What we can do is to reframe the impossible as what is possible and suggest future research. The exact algorithm can be used as a reference ("gold standard") for developing symheuristic models that are capable of providing similar outcomes but can be implemented with standard software tools.

Here is how this could be done. The exact CPA is applied to small portfolios where this is practically possible. The optimal global configurations of the portfolios are analyzed to identify patterns that are characteristic of the optimal solutions. The patterns serve to define rules (heuristics) that reduce the considered options, while the outcome is aimed at being close to that of the exact algorithm.

The heuristic model is combined with Monte Carlo simulation to obtain a simheuristic model that is applied to a greatly reduced number of configurations. The results of its application are compared with those of the full algorithm.

AI tools can dramatically speed up and improve the process of developing practical models by automating the iterative and incremental convergence of the outcomes of the exact CPA and the simheuristic CPA.

Conclusion

CPA provides optimal allocation of portfolio resources within the modeled decision space. In addition, it generates a portfolio plan that complies with all defined conditions and restrictions.

Although the need for huge computational resources limits its practical application, CPA can be used as a reference for developing practical simheuristic models.

Potentially, CPA can outperform any multi-step, even two-step, portfolio allocation algorithm. Even the simplified algorithm described in the first article about the CPA [1] has the potential for considerable reduction of resource waste compared to traditional allocation approaches.

The algorithm is applicable not only to project portfolios. It can be adapted for resource allocation in organizations, regions, countries, and other systems.

References

[1] Apostolov, A. (2025). The Combination-Permutation Algorithm: Optimizing Portfolio Allocation with TOC and Cost of Time, *PM World Journal*, Vol. XIV, Issue VII, July. Available online at https://pmworldlibrary.net/wp-content/uploads/2025/07/pmwj154-Jul2025-Apostolov-Combination-Permutation-Algorithm.pdf

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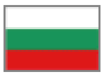

Alexander Apostolov holds a Master's degree in Economics and a PhD in Project Management and Sustainable Development. He has 25 years of experience in project management for new product development, construction, IT, events, and more.

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